Alma Mater Studiorum Università di Bologna
Department of Civil, Chemical, Environmental and Materials Engineering

A STOCHASTIC ANALYSIS OF THE NON-NEWTONIAN HYDRAULIC BEHAVIOUR OF ROUGH FRACTURES

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Session: MS 21 – (MS21) Non-linear effects in flow and transport through porous media

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FLOW OF SHEAR-THINNING FLUIDS IN GEOLOGICAL FRACTURES

The **hydraulic behavior of geological formations** is mainly governed by **fractures connectivity** and **permeability**, whose heterogeneity strongly affects flow and transport, with fluid rheology playing an important role often oversimplified.

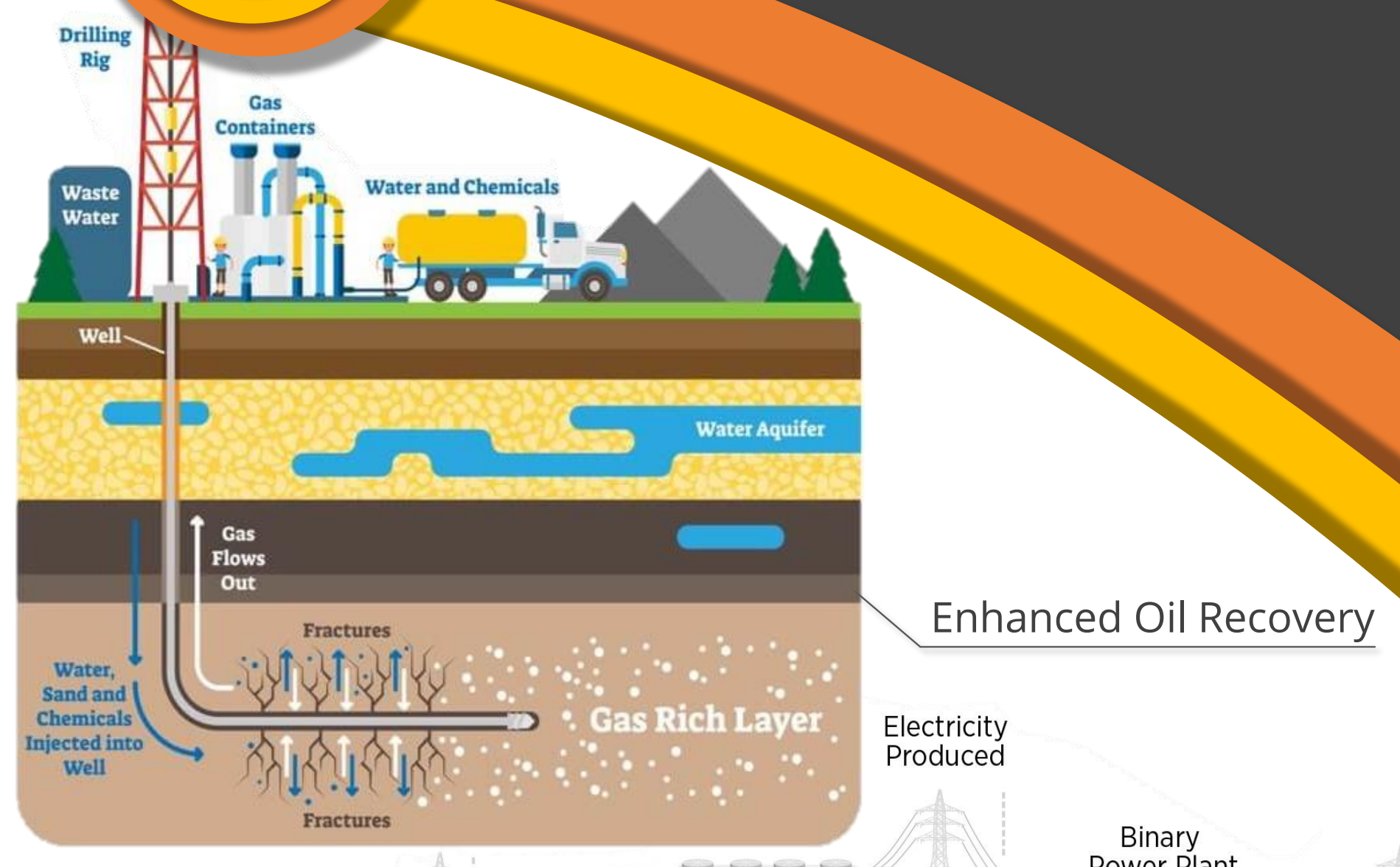
- Typically, unconventional and deep geotermal reservoirs present both **low-porosity** and **low-permeability**
- Activities in gas shale or hot rock requires **hydraulic stimulation** to enhances productivity and become cost effective
- Fluid viscosity is stryctly linked with the activity effectiveness (e.g., proppant transport or stimulation fluid recovery).

Fluids involved in subsurface industrial activities present a **shear-thinning behaviour** at **continuum scale,** due to their complex make-up.

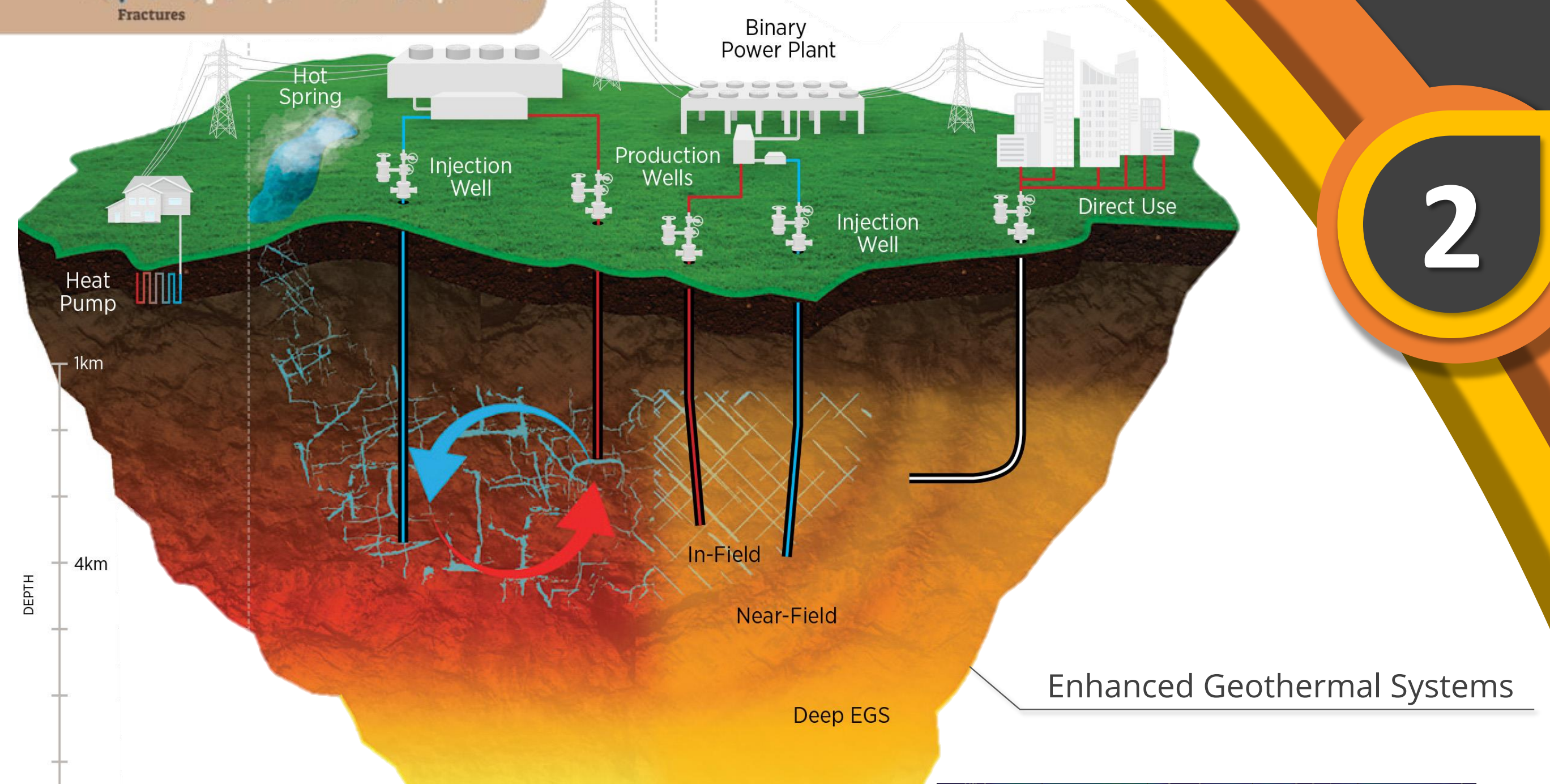
- muds
- foams
- water-based suspentions

Subsurface industrial activities:

- Enhanced Oil Recovery (EOL)
- Enhanced Geothermal Systems (EGS)
- Carbon sequestration
- Lost circulation of drilling fluids



Enhanced Oil Recovery



Enhanced Geothermal Systems

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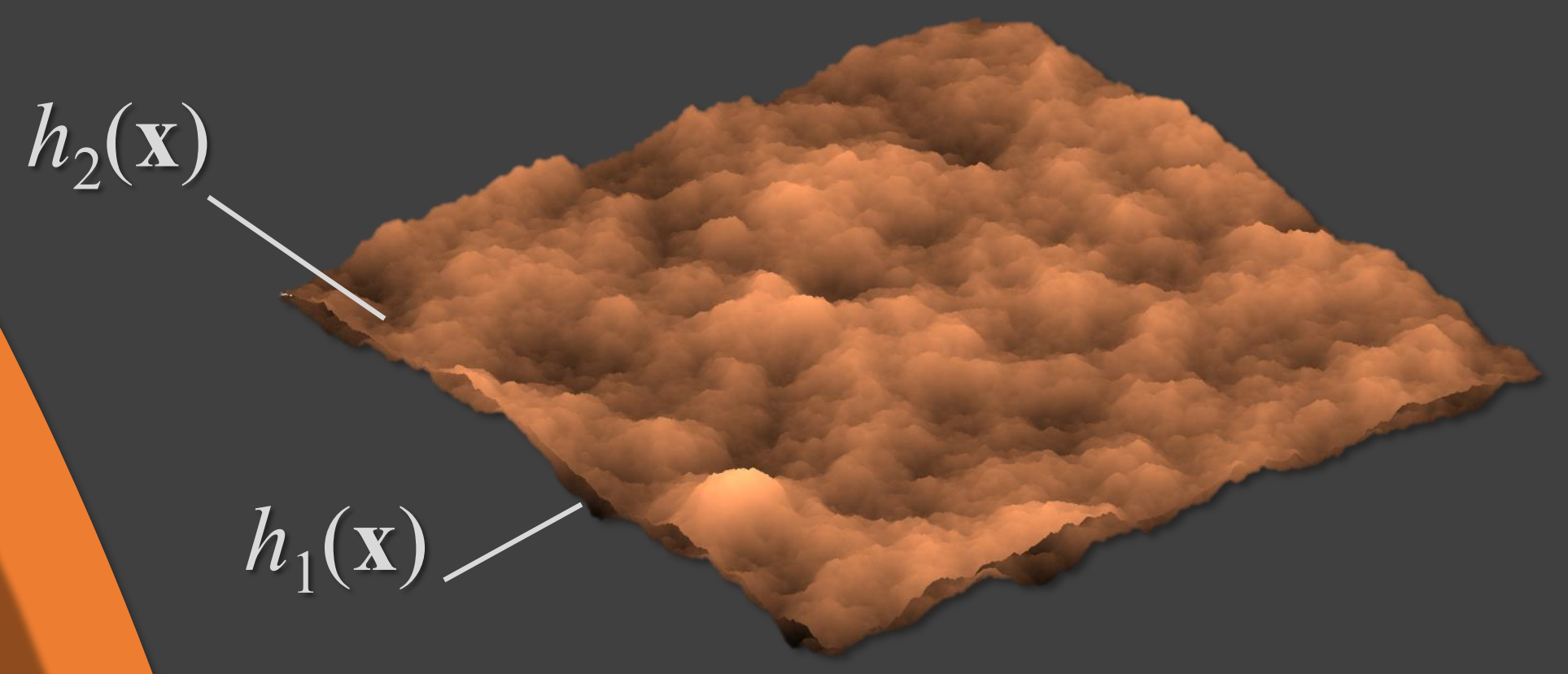
NUMERICAL MODELING



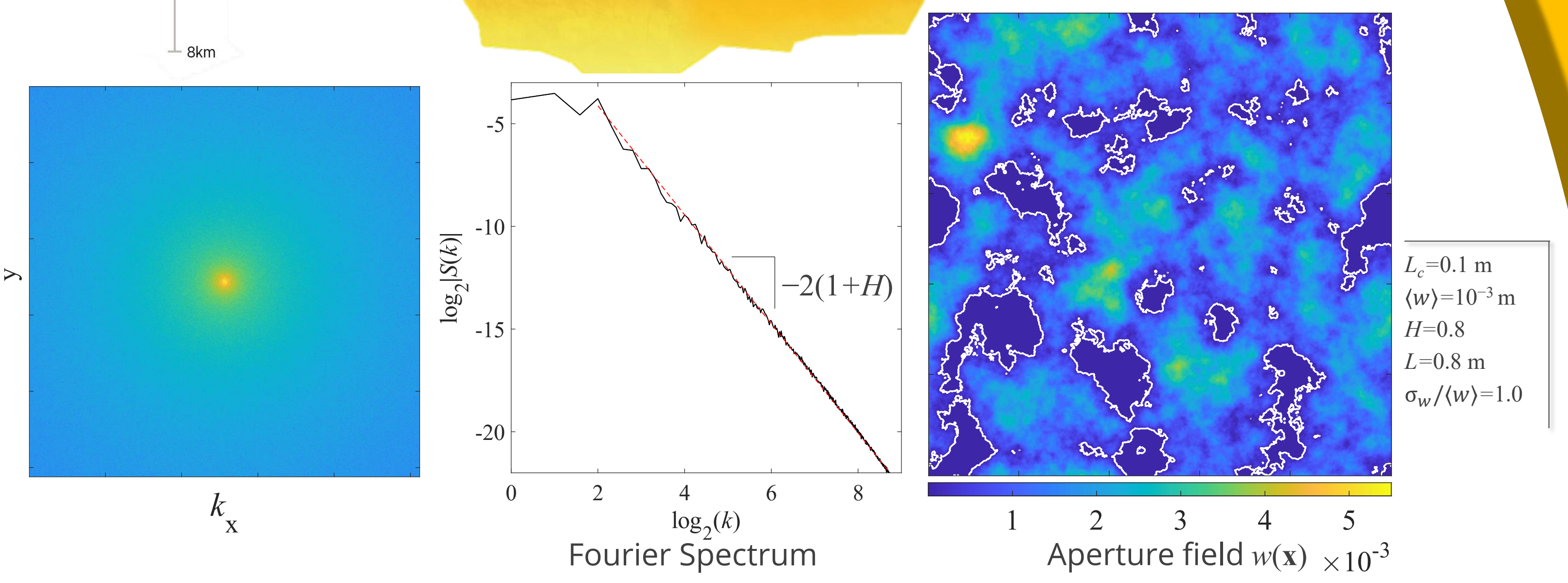
2.1 Synthetic fracture generator

A synthetic aperture field is estimated by mating two **isotropic self-affine surfaces** $h_i(\mathbf{x})$

$$w(\mathbf{x})=h_1(\mathbf{x})-h_2(\mathbf{x})+\langle w\rangle$$

A rough surface can be generated as a 2D white noises and introducing spatial correlation: multiplying the modulus of the Fourier transform by the modulus of the wave numbers $|k| =(k_x^2+k_y^2)^{1/2}$ to the power $-1-H$

$$|h(\mathbf{k})|\rightarrow|k|^{-1-H}\,|h(\mathbf{k})|$$



Fourier Spectrum

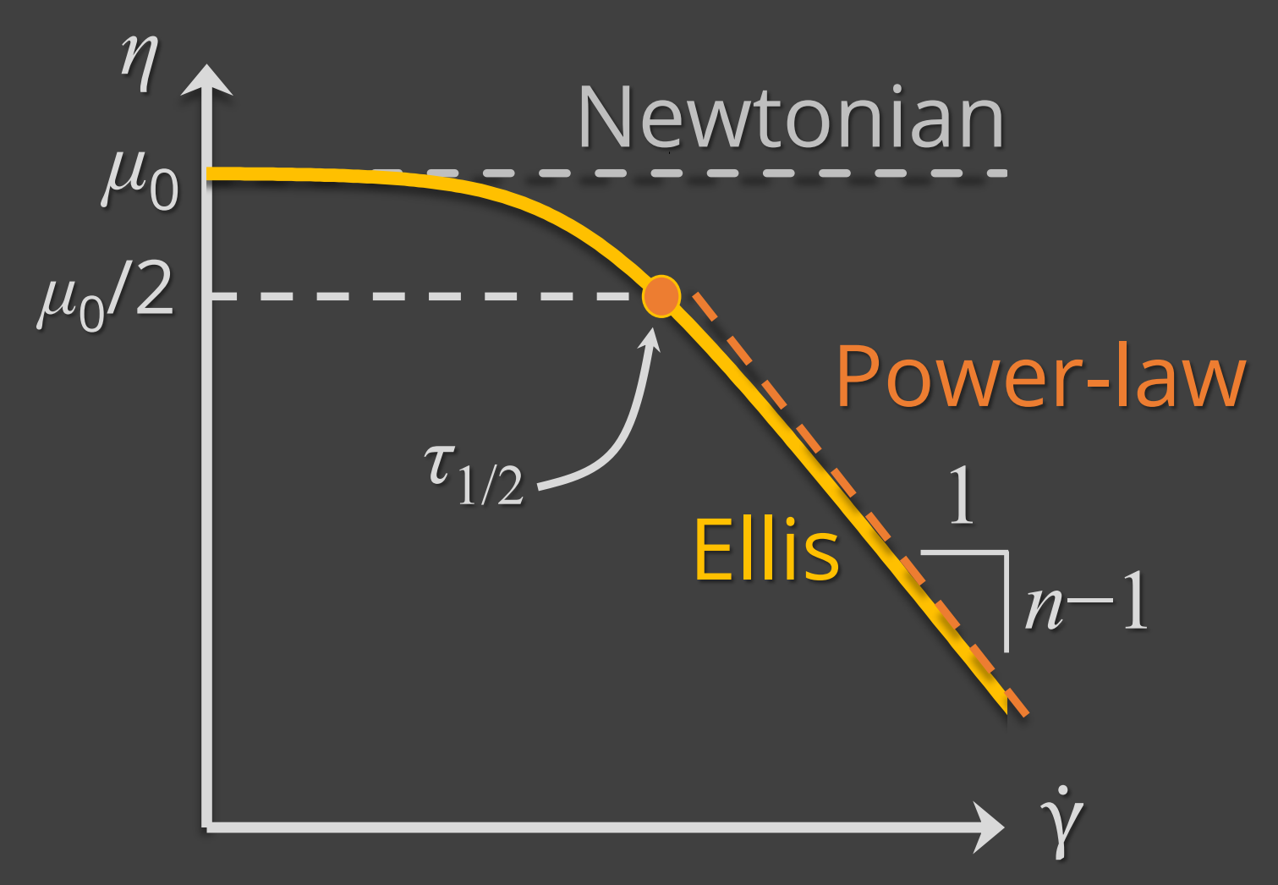
Aperture field $w(\mathbf{x})$ $\times10^{-3}$

2.2 Fluid Rheology: Ellis model

The Ellis model is a **three-parameter generalized Newtonian rheology**

$$\eta = \frac{\mu_0}{1+\left(\dfrac{\tau_{xz}}{\tau_{1/2}}\right)^{\frac{1}{n}-1}}$$

μ_0 low-shear rate viscosity ($\eta\rightarrow\mu_0$ for $\dot{\gamma}\rightarrow0$)
n shear-thinning index
$\tau_{1/2}$ characteristic shear stress: $\eta(\tau_{1/2})=\mu_0/2$



2.3 Generalized Non-linear Reynolds Equation

- **Lubrication theory** holds ($\nabla w\ll1$ and $\mathrm{Re}\ll1$)

$$-\nabla\cdot\left[\frac{w(\mathbf{x})^3}{12\mu_0}+\frac{n}{(2n+1)}\left(\frac{1}{2^{1+n}\mu_0^n\tau_{1/2}^{1-n}}\right)^{\frac{1}{n}} w(\mathbf{x})^{\frac{2n+1}{n}}|\nabla P|^{\frac{1}{n}-1}\right]\nabla P = 0$$

Numerical modeling via finite volume method:

- The non-linear system of equations is solve via **inexact Newton-Krylov** method
- **Variable-fill-in Cholesky preconditioned conjugate gradient** → linear problem
- A **parameters continuation strategy** is adopted to handle strongly non-linear cases

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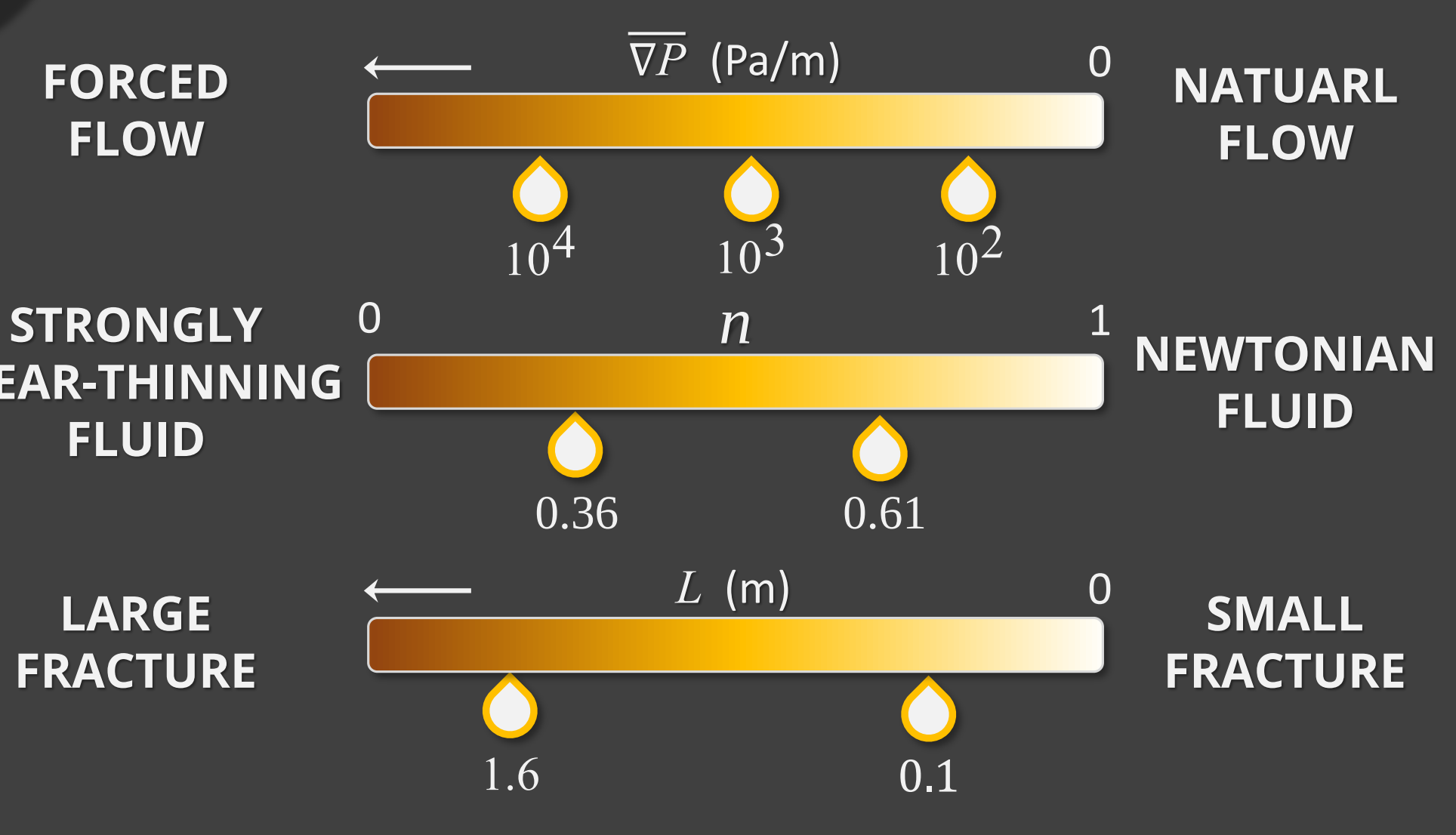
STOCHASTIC ANALYSIS

Parameter variability



FORCED FLOW ⟵ $\overline{\nabla P}$ (Pa/m) 0 **NATUARL FLOW**
10^4 10^3 10^2

STRONGLY SHEAR-THINNING FLUID 0 n 1 **NEWTONIAN FLUID**
0.36 0.61

LARGE FRACTURE ⟵ L (m) 0 **SMALL FRACTURE**
1.6 0.1

Fracture Geometry

$\sigma_w/\langle w\rangle$	0.8
$\langle w\rangle$	10^{-3} m
H	0.8
L_c	0.1 m

Fluid Rheology		n (-)	μ_0 (Pa·s)	$\tau_{1/2}$ (Pa)
Silicon Oil	**SO**	0.61	9.75	9.23
Xanthan Gum	**XG**	0.36	4.42	0.199

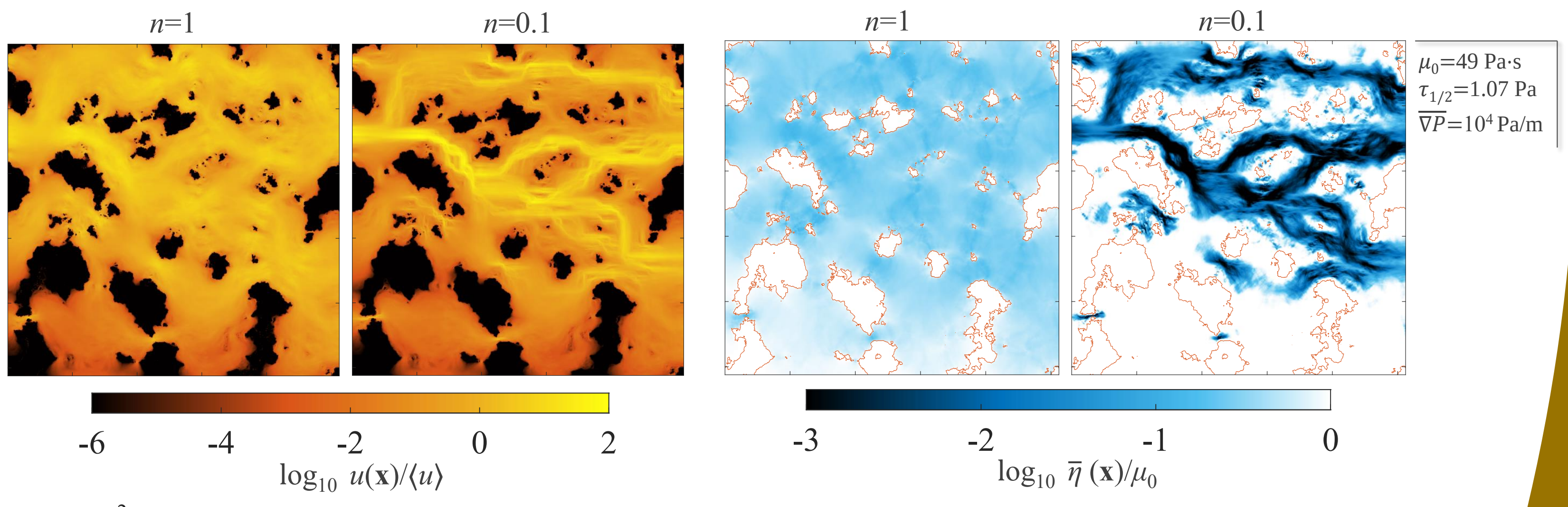
A **Monte Carlo Framework** is adopted to study the impact of:

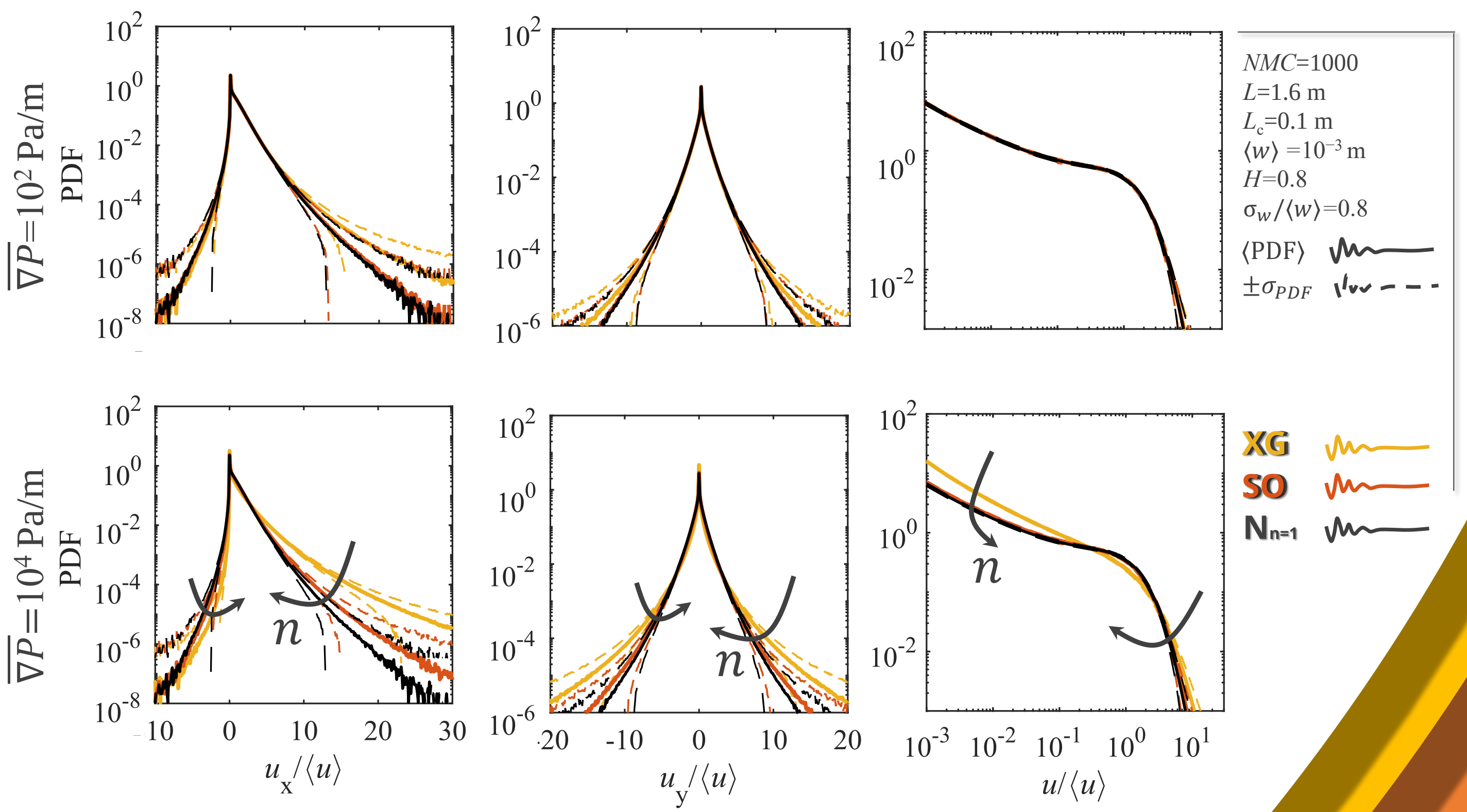
- **Pressure gradient** ($\overline{\nabla P}$) → Flow regime
- **Shear-thinning behaviour** (n) → Fluid rheology
- **Fracture dimension** (L) → Scale

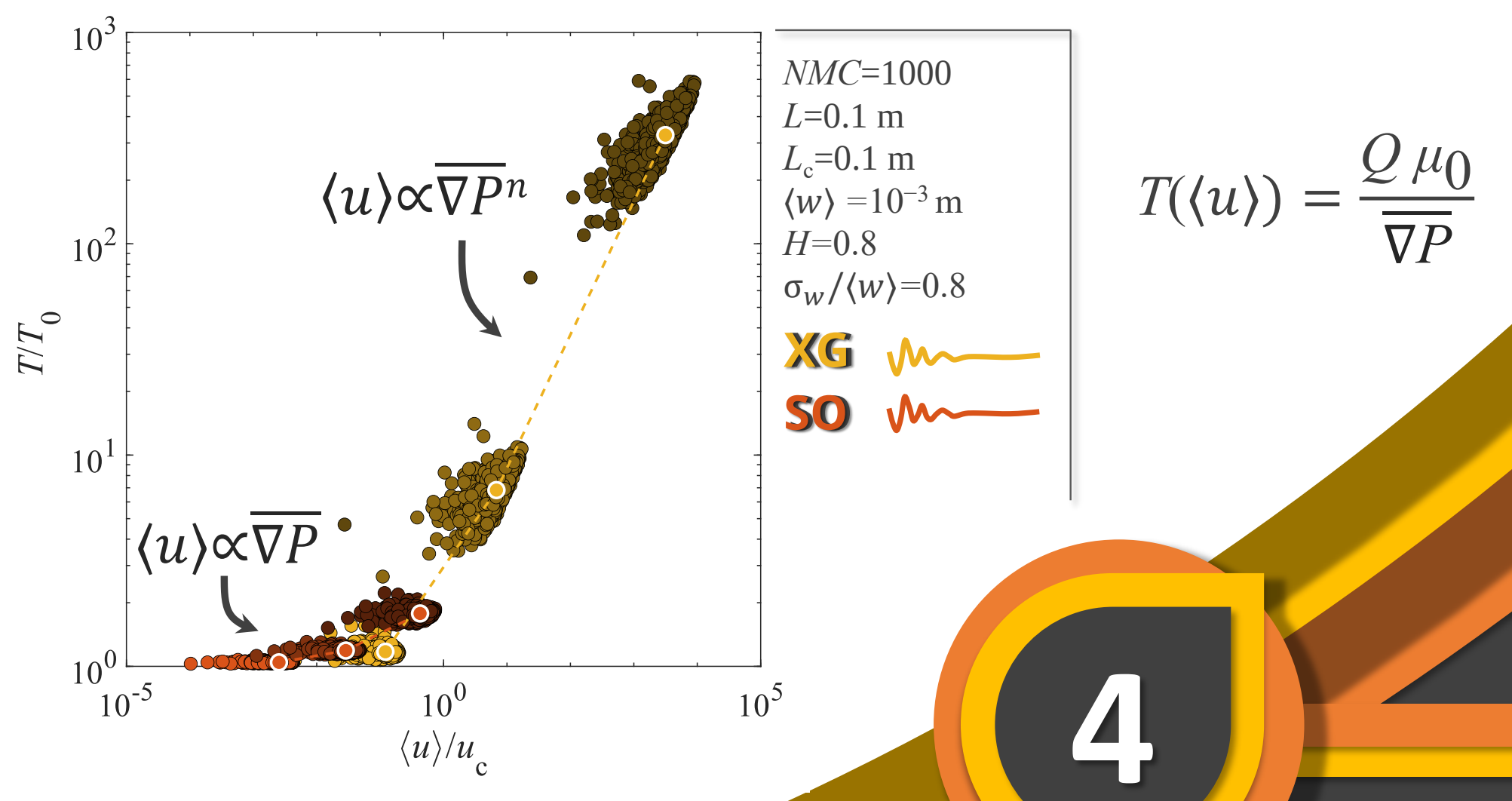
For each combination of the parameters (12 in total), the flow is solved considering a set of **1000 fracture realizations.**

Eventually, ensemble statistics are then analysed for:

- **Velocity** components and magnitude (u_x, u_y, u)
- **Apparent Transmissivity** (T)







$$T(\langle u\rangle) = \frac{Q\,\mu_0}{\overline{\nabla P}}$$

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CONCLUSION AND PERSPECTIVES

4.1 Conclusion

- Velocity components displays narrow PDFs with nearly exponential decay
- High $\overline{\nabla P}$ and low n incresase shear-thinning behaviour, favouring flow localization
- QoIs PDFs of the smaller fracture length present higher dispersion
- Shear-thinning behaviour enhances fracture transmissivity, leading to non-darcian flow regime $\langle u\rangle\propto\overline{\nabla P}^n$

4.2 Perspectives

Flow localization impacts the effectiveness of any subsurface activities

- **High velocities** magnitude PDFs → **channeling metrics**
- **Low velocities** magnitude PDFs → **anomalous transport**

Mèheust, Y., Schmittbuhl, J. (2000) Flow enhancement of rough fracture. *Geophys. Res. Lett.*, 27(18), 2989-2992. doi: 10.1029/1888gl1008464

Lenci, A., Mèheust, Y., Putti, M., Di Federico, V., (2021) Shear-thinning hydraulic behaviour of rough fractures: a stochastic analysis via Monte Carlo simulations, *Water Resour. Res.* (under submition)

Abstract ID 298